

Math program notes

If we see a tree, we enjoy its shape, size, colors, smells, and tastes with our senses; and each of these attributes can be expressed in the abstract language of mathematics. Most of us do not tend to experience the phenomena of our lives – the rhythm of our heartbeats, the proportions of our body, the shape of our house, the ratio of salt to soup, in mathematical terms. But we could, and some people do. There are people who have a sense for numbers, people for whom seven has an indefinable “seven-ness” that is different from all other numbers. Blind people whose sight is corrected for the first time as adults often see only two-dimensional shapes, while a select few are able to visualize the 11 dimensions of string theory. Doctors, biologists, physicists – scientists of all varieties, are able to see things in more quantifiable ways than the rest of us; and artists often have an intuitive grasp of pleasing proportion.

The much-touted “Mozart Effect” postulates that people who listen to music do better in logical thinking and mathematical tasks. There are several studies demonstrating that children who study music in preschool outshine their non-musical classmates in math. I believe it is because they have become not just intellectually but physically aware of mathematical relationships. In his book, *The Web of Life*, Fritjof Capra says, “Understanding of pattern is crucial to understanding the living world around us; and all questions of pattern, order, and complexity are essentially mathematical.” Music is comprised of layers upon layers of patterns; so, if you like, this concert can be a sort of “Where’s Waldo?” game.

We originally set out to try to explore and explain the relationship between math and music, but the task proved to be Herculean. A thousand programs’ worth of material would only scratch the surface. So what we are going to do is take ideas from two of our earlier programs, the one about the Five Senses and the one about Classical and Romantic worldviews, just a little further. We will experience the abstractions of mathematical pattern and proportion via our senses, aurally and visually. And we will experience the concepts of pattern and proportion as they unfold in time, something we only mentioned in passing as it relates to classical and romantic values.

About the music:

Epitaph of Seikilos, transcribed and edited by Thomas J. Mathiesen
(The following information is from *Apollo’s Lyre* by Thomas J. Mathiesen and *The Norton Anthology of Western Music, Ancient to Baroque*, ed. Claude Palisca)

Translation: As long as you live, shine
Grieve you not at all
Life is of brief duration
Time demands its end.

This drinking song was inscribed on a tombstone in the first century. A rubbing taken shortly after it was discovered reveals that it was originally 13 lines long, though the bottom line was later ground off to make it stand upright to be used as a plant stand! Lines 6-11 had musical notation above the text, including indications for rhythm. The piece serves as a sort of embodiment of the musical proportions considered ideal at the time. The piece uses each note of the scale e-e', with f# and c#. To quote Dr. Mathiesen: "the opening interval is a fifth, the structural points of the first and fourth lines outline a fourth, the overall range is an octave, and there are several other similar proportions; in addition, the poem is in iambic dimeter (Greek and Latin grammarians regularly observed the proportional relationships in rhythms and meters)."

Webern, *Entflieht auf leichten Kähnen*

Anton Webern is best known as a disciple of the 12-tone method of composition pioneered by Arnold Schoenberg. This piece does not use that method, but, like the Epitaph and the Ockeghem, it seems to systematically employ the full range of useable intervals (at this point a much larger number of them were considered useable than in Seikilos' time). Like the Ockeghem, it uses a series of double canons, and it is constructed in a clean arch form.

Translation:

escape on light boats waiting
- fairy worlds of sunlight –
so that your tears' abating
repays you for your flight

look at this tangled blondness
a sky blue dream made bold
and watch the drunken bliss
without delight unfold

may such a lovely shiver
not clothe you in new pain
but quiet grief be giver
of fullness to spring again

Stefan George, translated by Marie Deer

Nun danket alle Gott chorale by Martin Rinkart (words) and Johann Crüger (music).

Johannes Ockeghem, *Missa Prolationem*

Kyrie I - Christe eleison - Kyrie II

Ockeghem's *Prolation Mass* is a masterpiece of compositional organization. Each movement is a double canon (the sopranos and altos sing in canon, and the tenors and basses sing in canon – though the *Christe* does not really sound like a canon, because the sopranos and tenors do not begin their statement until the altos and basses finish theirs). Furthermore, Ockeghem systematically exploits the intervals of the scale in the intervals of the canons: the Kyrie I canons are at the unison, the *Christe* canons at the second, the Kyrie II canons at the third, the Gloria canons at the fourth and so on. The Mass derives its name from the fact that the soprano, alto, tenor, and bass are each composed in a different “prolation” or rhythmic notation. The four prolations are equivalent to today's $3/4$, $2/4$, $9/8$, and $6/8$ time. Ockeghem would write out a line of music and instruct, for example in Kyrie II, the sopranos to sing the line in $3/4$ time and the altos to sing it in $2/4$ starting a third lower, miraculously deriving two compatible lines from one. Then he repeats the process for the tenors and basses, and combines all four parts. One might say the “structure” of this piece is not a static form but rather a series of processes.

Guillaume Dufay, *Nuper rosarum flores*

Dufay wrote the motet *Nuper rosarum flores* in 1436 for the dedication of the Cathedral della Santa Maria del Fiore in Florence. It has traditionally been believed to reflect the architectural proportions of the cathedral and its famous dome by the architect Brunelleschi, and certain Pythagorean proportions are certainly represented in both. But an influential article from 1993 by Craig Wright theorizes that the piece more accurately reflects the mystical numbers of Solomon's Temple, which is invoked repeatedly in the consecration ceremony. According to Wright: “The Bible relates that work was begun in the fourth year of Solomon's reign, that it took seven years to complete, that it was dedicated in the seventh month of the year, and that the service of dedication required twice times seven days.” In addition, “the dimensions of Solomon's temple, reduced to their lowest common denominator, were 6 (length), 4 (length of nave), 2 (length of sanctuary and width of building), and 3 (height of building).

The piece is both structured according to mathematical proportions and saturated with number symbolism, particularly using the numbers 7 and 4. The text consists of 4 stanzas of 7 lines, 7 syllables each. The cantus firmus always starts 7 duplex longae (we would call it 14 measures) after the upper duets. The piece is in 4 sections: section one is (in modern terms) 56 measures in $3/2$, section two is 56 measures in $4/4$, section three is 28 measures in $4/4$, and section 4 is 28 measures in $6/4$. (In the old notation, each section consists of $28 + 28$ breves in each of the four prolations). So the relative proportions of the four sections work out to 6:4:2:3, reflecting both the dimensions of the Biblical temple (scrolled out in time) and the divine proportion of 3:2 (reflecting the ideal proportions of a dome, as exemplified by Brunelleschi's and also the dome of the Pantheon). The double cantus firmus may also reflect the double-shell construction which made the huge size of the cupola possible. The 3:2 proportion also creates the

Perfect 5th, which is reflected in the cantus firmus lines, which proceed in canon at the fifth and often sing harmonic fifths, as well.

Johann Sebastian Bach, Cantata No. 192, *Nun danket alle Gott*

According to Tushaar Power's 2001 Duke University Ph.D. thesis, *J.S. Bach and the Divine Proportion*, Bach's Cantata 192 was structured sequentially according to the principles of the Golden Section: "He would have taken the given length of the first movement, 169 measures, and multiplied this by 5:3. This produced a value of $281\frac{2}{3}$, which he then rounded down to 281, the number of measures comprising movements 1 and 2. Subtracting the given length of the first movement from the number of measures comprising the first two movements, he would have determined that the second movement would be 112 measures long; i.e. $281 - 169 = 112$. Thus, the composer constructed a two movement complex which is divided at the major Golden Section." He then repeated the process to derive the length of the third movement, except that this time he used the ratio 13:8, which is higher up on the Fibonacci series and more closely approximates the "Divine Proportion". So there are three movements, but two structural constructs, each comprising two movements. The second movement becomes the smaller half of the Golden Section comprised of Movts. 1 and 2 and the larger half of the Golden Section comprised of Movts. 2 and 3.

Bach, like Dufay, may have structured the piece mathematically while also using numbers with symbolic value. Though the first movement of the piece has 169 measures, the last measure has only one harmonically necessary beat. If Bach considered the movement to have 168 full measures of music, then the lengths of the three movements in measures (168, 112, 70) can all be factored by 14. The number 14 is the alpha-numerical representation of "Bach" (if you number the alphabet, the letters B-A-C-H are 2+1+3+8). (This information is also from Powers' thesis).